Flutter analysis of bridges with curvature in plan view

José Ángel Jurado[1], Juan Quintela[2], Santiago Hernández [3], Ibuki Kusano[4]

[1]*School of Civil Engineering, Universidade da Coruña, Spain, jose.angel.jurado@udc.es*
[2]*School of Civil Engineering, Universidade da Coruña, Spain, juan.quintela.suarez@udc.es*
[3]*School of Civil Engineering, Universidade da Coruña, Spain, hernandez@udc.es*
[4]*IQS School of Engineering, Universitat Ramon Llull, Spain, ibuki.kusano@iqs.edu*

SUMMARY:
Most of the long span bridges follow a straight alignment for the deck geometry. This characteristic does the building of the aeroelastic matrices during the multimodal flutter analysis simple. Moreover, if the bridge deck has constant section, all the elemental aeroelastic matrices will be equal and the global structural matrices can be easily built. However, there are cases of slender cable supported bridges with curvature in plan view, especially footbridges. The geometry of these structures present difficulties for the matrix analysis because the different coordinate system of each deck element of the structural model. The software developed by the University of La Coruña for the flutter aeroelastic analysis was initially prepared for straight bridge decks with constant cross section. Therefore it has been modified for the studies of non-straight deck arrangements. This paper shows the changes in the analysis formulation and explains how to consider the curved geometry and the inclination of the main wind direction with respect to the deck axis. An example of a curved suspension footbridge is presented using the formulation.

Keywords: flutter, long span bridges, curvature in plan view

1. AEROELASTIC ANALYSIS OF CURVATURE PLAN BRIDGES

Nowadays the design of cable supported bridges, in particular footbridges, often presents sophisticated geometries with curved shapes, which provide beautiful views and pleasant sensations while crossing them. The slender and flexible elements of this kind of structures may cause wind instabilities which requires aeroelastic analysis. Examples of this kind of structures are Gateshead Millennium Bridge in UK, Pont de Térénez in France, or Ponte de Mare in Italy (see Fig.1).

Figure 1. Curved deck bridges.

In 2001 the University of La Coruña programed a multi-modal analysis software to assess the wind flutter speed of long span bridges. This code was used in multiple researches for the optimization of long span bridges as Jurado & Hernandez 2004, Jurado et al. 2011 or Kusano et al. 2018. In 2013 the routines were improved to solve problems associated to repeated eigenvalues (Jurado et al. 2013). In 2019 it was modified to analyse cases where the aerodynamic behaviour along the bridge span is not constant, for example in cases of an increasing height of the deck cross section. Recently, the project of a curved suspension footbridge has caused the modification explained in this work. Starting with the dynamic equilibrium equation

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{f}_a \tag{1}$$

where **M**, **C**, and **K** are global mass, damping, and stiffness matrices respectively, $\mathbf{u}, \dot{\mathbf{u}}, \ddot{\mathbf{u}}$ are displacement, velocity and acceleration vector of the structure. $\mathbf{f}_a$ can be expressed as the product of an aeroelastic stiffness matrix $\mathbf{K}_a$ by the displacements plus an aeroelastic damping matrix $\mathbf{C}_a$ multiply by the velocities. These matrices should be assembled considering all the elements of the deck discretization, but on the global structural coordinate system. The forces acting on deck structural nodes are related to its movements and velocities. Thus, for either element end i = 1, 2 the following matrix expression can be written in local coordinate system:

$$\mathbf{f}'_{a,i} = \mathbf{K}'_{a,i}\mathbf{u}'_i + \mathbf{C}'_{a,i}\dot{\mathbf{u}}'_i =$$
$$\frac{1}{2}\rho U^2 K^2 \frac{l_e}{2}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & P_4^* & -P_6^* & -BP_3^* & 0 & 0 \\ 0 & -H_6^* & H_4^* & BH_3^* & 0 & 0 \\ 0 & -BA_6^* & BA_4^* & B^2A_3^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}\begin{Bmatrix} u'_i \\ v'_i \\ w'_i \\ \varphi'_{xi} \\ \varphi'_{yi} \\ \varphi'_{zi} \end{Bmatrix} + \frac{1}{2}\rho U^2 \frac{KB}{U}\frac{l_e}{2}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & P_1^* & -P_5^* & -BP_2^* & 0 & 0 \\ 0 & -H_5^* & H_1^* & BH_2^* & 0 & 0 \\ 0 & -BA_5^* & BA_1^* & B^2A_2^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}\begin{Bmatrix} \dot{u}'_i \\ \dot{v}'_i \\ \dot{w}'_i \\ \dot{\varphi}'_{xi} \\ \dot{\varphi}'_{yi} \\ \dot{\varphi}'_{zi} \end{Bmatrix} \tag{2}$$

where l_e is the element length, p =(1/2)ρU^2, B is the deck with, ρ the air density, U the perpendicular wind speed, K = $B\omega/U$, reduced frequency with ω frequency response and $H^*_i(K)$, $P^*_i(K)$, $A^*_i(K)$ i = 1...6 are the flutter derivatives. In order to transform the matrices to the global coordinate system it is necessary to operate with a matrix built with cosines of angles between the element and global axes. For example α_y is the angle between the y element direction and the x global axis, or β_x is the angle between the x local direction and y global axis, etc.

$$\mathbf{L} = \begin{pmatrix} \cos\alpha_x & \cos\alpha_y & \cos\alpha_z & 0 & 0 & 0 \\ \cos\beta_x & \cos\beta_y & \cos\beta_z & 0 & 0 & 0 \\ \cos\gamma_x & \cos\gamma_y & \cos\gamma_z & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\alpha_x & \cos\alpha_y & \cos\alpha_z \\ 0 & 0 & 0 & \cos\beta_x & \cos\beta_y & \cos\beta_z \\ 0 & 0 & 0 & \cos\gamma_x & \cos\gamma_y & \cos\gamma_z \end{pmatrix} \tag{3}$$

Zero positions are due to the translations or forces do not participate on the rotations or moments and vice versa. The local aeroelastic matrices on the global coordinate system are calculated by

$$\mathbf{K}_{a,i} = \mathbf{L}\mathbf{K}'_{a,i}\mathbf{L}^{T} \ ; \ \ \mathbf{C}_{a,i} = \mathbf{L}\mathbf{C}'_{a,i}\mathbf{L}^{T} \tag{4}$$

By applying modal analysis, the deck displacements can be expressed as a linear combination of most relevant m vibration modes grouped in the modal matrix $\mathbf{\Phi}$, $\mathbf{u} = \mathbf{\Phi}\mathbf{q}$, where $\mathbf{q}$ is the participation vector. Equation (1) is reduced to

$$\mathbf{I}\ddot{\mathbf{q}} + \mathbf{C}_{R}\dot{\mathbf{q}} + \mathbf{K}_{R}\mathbf{q} = \mathbf{0} \tag{5}$$

Where $\mathbf{I} = \mathbf{\Phi}^{\mathbf{T}}\mathbf{M}\mathbf{\Phi}$ is a unit matrix if the vibration modes are normalized with respect to mass, $\mathbf{C}_{R} = \mathbf{\Phi}^{\mathbf{T}}(\mathbf{C}\text{-}\mathbf{C}_{a})\mathbf{\Phi}$ and $\mathbf{K}_{R} = \mathbf{\Phi}^{\mathbf{T}}(\mathbf{K}\text{-}\mathbf{K}_{a})\mathbf{\Phi}$. The system of equation (5) can be transformed in an eigen-problem which solution is a set of m couples of complex eigenvalues, where the real part is related with the damping of the response and the imaginary part is the frequency. The transition from positive to negative damping determines the flutter instability.

Additionally, the value of the perpendicular wind velocity on the elements changes along the deck because the curvature in plan view. Therefore, for a correct analysis, it is necessary on each element to multiply the main speed by the cosine of the angle between the wind direction and the longitudinal deck axis.

2. APPLICATION TO A CURVED SUSPENSION FOOTBRIDGE

A slender suspension footbridge of curvature plan deck is under construction over the Miño River between Portugal and Spain. The single span length is 300 m and it has one eccentric main cable with respect to the curved deck. There are two pylons at either end of the structure. The cross section of the deck is triangular of 4 m width. The unusual design of this slender structure showed in Fig. 2 required aeroelastic flutter analysis using the software of the University of La Coruña modified to include the formulation explained in the previous section.

Sets of flutter derivatives for the triangular deck section were obtained testing a sectional model in a wind tunnel. The experiments were performed considering three angles of attack: -2, 0 and +2. The natural frequencies and modal shapes were assessed using a non-lineal structural model with SAP2000 (see Fig. 2). Finally two multimodal flutter analyses were carried out with 20 vibration modes. One with the empty deck without barriers to simulate the construction phase, and another one considering the finished footbridge.

Table 1 shows the critical wind velocity results U_{crit} of the suspension footbridge with and without barriers. The values are very similar for the three angles of attack. As expected, the results without barriers give greater flutter speed because of the better aerodynamic behavior and lower structural mass, which makes the natural frequencies increase.

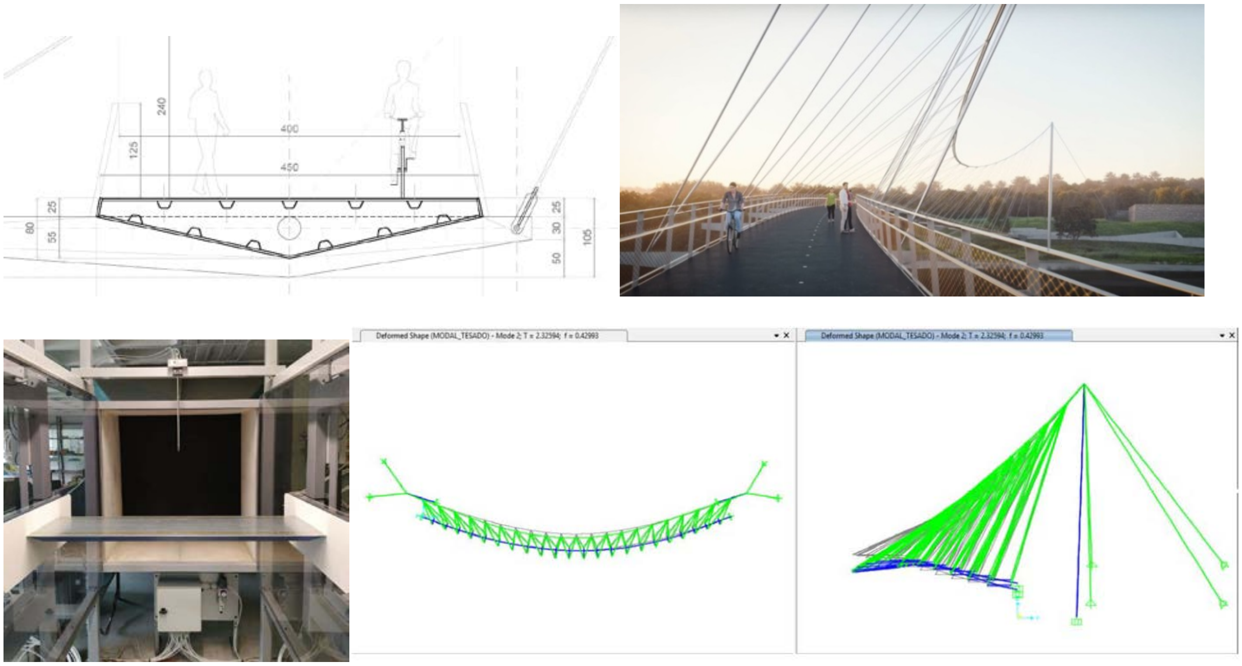

Figure 2. Deck cross section, virtual view of the footbridge, sectional model in the wind tunnel and SAP2000 dynamic analysis.

Table 1. Flutter speed results.

	Angle of attack	U_{crit} (m/s)
Without barriers	0^o	79.80
	$+2^o$	79.18
	-2^o	80.98
Completed footbridge	0^o	63.84
	$+2^o$	63.67
	-2^o	64.17

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